Journal of Statistical Physics, Vol. 80, Nos. 3/4, 1995

Phase Transitions in a Driven Lattice Gas at Two Temperatures

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Received December 15, 1994

By suitably combining the uniformly driven lattice gas and the two-temperature kinetic Ising model, we obtain a generalized model that allows us to probe a variety of nonequilibrium phase transitions, including a type not previously observed. This new type of transition involves "longitudinally ordered" steady states, which are phase-segragated states with interface normals *parallel* to the drive. Using computer simulations on a two-dimensional lattice gas, we map out the structure of the phase diagram, and the nature of the transitions, in the three-dimensional space of the drive and the two temperatures. While recovering anticipated results in most cases, we find one surprise, namely, that the transition from disorder to longitudinal order is continuous. Unless it turns out to be very weakly first order, this result is inconsistent with the expectation of field-theoretic renormalization group calculations.

KEY WORDS: Driven diffusive system; nonequilibrium steady states; lattice gas; Monte Carlo simulations; phase transitions.

1. INTRODUCTION

Phase transitions associated with non-equilibrium steady states are much more complex than those that occur in equilibrium, and have recently received a lot of attention.[2] Systems exhibiting such transitions are driven away from equilibrium by various external forces and settle into a time-independent state whose probability distribution is not simply given by the usual Boltzmann factor, but also depends on the details of the dynamics which control the evolution.

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[2] See, e.g., Ref. 1 for review of a variety of nonequilibrium systems such as self-organized criticality, kinetic roughening of interfaces, etc.

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0022-4715/95/0800-0499$07.50/0

One simple example of such a system is the driven diffusive Ising model[2] (see Ref. 3 for a recent review), which consists of an ordinary lattice gas with nearest-neighbor interactions evolving with Kawasaki dynamics[4] that are biased by an external field **E** so that particles (holes) prefer to move along (against) the field. Imposing *periodic boundary conditions*, we find that the system eventually settles into a nonequilibrium stationary state with a nontrivial steady particle current. Like its equilibrium counterpart, this model undergoes a continuous order–disorder transition. At high temperatures, the system is in a disordered, homogeneous phase, while at low temperatures, it is in an inhomogeneous phase, where particle-rich and particle-poor regions coexist. However, unlike the equilibrium Ising model, in this model the interfaces between such regions are invariably aligned "parallel to the field," i.e., their normal vectors are orthogonal to the field.

Two closely related models are (i) the above system driven with an *annealed random* field[5] and (ii) the diffusive, two-temperature kinetic Ising model.[6–8] They are believed to be so similar that, for simplicity, we will focus only on one version: the latter. Here, one of the d dimensions of a hypercubic lattice is singled out and referred to as the "parallel" or "longitudinal" direction, while the other $d-1$ directions are labeled "perpendicular" or "transverse." On this lattice, we place an ordinary Ising model with isotropic and uniform nearest-neighbor interactions. The novelty lies in the dynamics, which involves Kawasaki exchanges coupled to two different heat baths that in general are at different temperatures. Specifically, particle moves in the parallel direction are governed by transition rates appropriate for being in contact with a reservoir at temperature $1/\beta_{\parallel}$, while moves in perpendicular directions are controlled similarly by an inverse temperature $\beta_{\perp}$. If $\beta_{\parallel}=\beta_{\perp}$, the model reduces to the familiar Ising lattice gas, and eventually settles into the equilibrium state. However, if $\beta_{\parallel}\neq\beta_{\perp}$, the system experiences an energy flux, from the bath with higher temperature to the lower one, and thus settles into a nonequilibrium steady state. As a result, neither time reversal nor detailed balance holds in this type of stationary state, even if the nearest-neighbor interactions were anisotropic. For small β's, this state is homogeneous and disordered. At large β's, the system phase segragates into lattice-spanning regions of high/low particle concentration in one of two ways, distinguished by the orientation of the interfaces between such regions. Specifically, the *normal* of the interfaces will lie in the subspace corresponding to the larger β. Referring to the phase diagram[7] in the $\beta_{\perp}$–$\beta_{\parallel}$ plane (Fig. 1), we describe these two phases as "transverse order" if $\beta_{\perp}>\beta_{\parallel}$ and "longitudinal order" if otherwise. The transitions between the disordered and the ordered phases are continuous, while between the two ordered states, the transition is

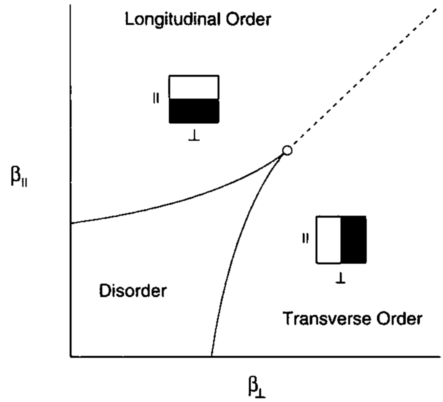

Fig. 1. Schematic phase diagram of the $d=2$ diffusive two-temperature Ising model.[6] Two different ordered phases, shown schematically in the insets, are separated by a line of first-order transitions (indicated by the dashed line). Between each of these and the disordered phase are lines of continuous transitions (indicated by solid lines). All phases meet at a bicritical point (indicated by the open circle), which corresponds to Onsager's critical point.

first-order. Both of the critical lines meet the first-order line at the Onsager point of the equilibrium Ising model (indicated by ○ in Fig. 1), which naturally leads to its being labeled a *bicritical point*.

In this paper, Monte Carlo simulations are used to study the effects of combining these two type of drives on the Ising lattice gas. Apart from the intrinsic interest in the three-dimensional ($\mathbf{E}$, $\beta_{\perp}$, $\beta_{||}$) phase diagram, we are motivated to find a microscopic lattice dynamics for testing a model first studied within the context of continuum field theories for which a first-order transition was predicted.[9]

The remainder of this paper is organized as follows. In the next section, we review the predictions of dynamic renormalization-group field theories, with the main focus on the case mentioned in the previous paragraph. In Sec. 3, we introduce the driven lattice gas, coupled to two different thermal baths, examine its relationship with the field-theoretic model, and discuss the details of the simulations. The results of the simulations are then described in Sec. 4. Finally, Sec. 5 contains a summary and suggestions for further study.

2. REVIEW OF CONTINUUM FIELD-THEORETIC MODELS

Soon after the standard driven lattice gas[2] was introduced, a dynamic field-theoretic description[9,10] of its second-order phase transition was proposed. The starting point is the Langevin equation for an Ising model with a conserved dynamics, i.e., Model-B in the language of Halperin and Hohenberg[11]

$$\frac{\partial \phi}{\partial t} = \lambda \nabla^2 \frac{\delta \mathscr{H}}{\delta \phi} + \eta \tag{1}$$

In this equation, $\phi(\mathbf{x}, t)$ is a coarse-grained scalar order parameter, with

$$\mathscr{H}[\phi] = \int \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} \tau \phi^2 + \frac{1}{4!} g \phi^4 \tag{2}$$

being a Landau–Ginzburg–Wilson Hamiltonian. The effects of the conserved noise are modeled by $\eta(\mathbf{x}, t)$, a Gaussian with zero mean and

$$\langle \eta(\mathbf{x}, t)\, \eta(\mathbf{x}', t') \rangle = 2\lambda \delta(t - t')\, \nabla^2 \delta(\mathbf{x} - \mathbf{x}')$$

The temperature dependence of the model is contained most essentially in τ, with $\tau \to 0$ as the signal for criticality.

When an external drive is added to the system, (2) must be modified accordingly. The most obvious difference is the presence of an ohmic current which is set up in the steady state. Since Eq. (1) is in the form of a continuity equation, we may simply add a term to the right-hand side of the form $-\nabla \cdot \sigma(\phi) \mathbf{E}$, with $\sigma(\phi)$ being a density-dependent conductivity. Taking into account the excluded-volume constraint and writing a coarse-grained parameter for the drive (in place of the microscopic E), we find that the extra term is $\mathscr{E} \partial \phi^2$, where ∂ indicates a gradient operator in the direction of the drive. Another important effect was then recognized,[9,10] namely, that such a term renormalizes only those couplings in (1) which involve ∂. As a result, it is vital to replace (1) by a fully anisotropic version. The final equation is

$$\begin{aligned}\frac{\partial \phi}{\partial t} = \lambda \bigg[&(\tau_\perp \nabla_\perp^2 + \tau_\parallel \partial^2) \phi - (\alpha_\perp \nabla_\perp^4 + 2\alpha_\times \partial^2 \nabla_\perp^2 + \alpha_\parallel \partial^4) \phi \\ &+ \frac{1}{3!} (g_\perp \nabla_\perp^2 + g_\parallel \partial^2)\, \phi^3 \bigg] + \mathscr{E} \partial \phi^2 + \eta_\perp + \eta_\parallel \end{aligned} \tag{3}$$

where $\nabla_\perp$ indicates the gradient operator for the subspace transverse to the field. Note that the noise term has also become anisotropic, with correlations

$$\langle n_\perp(\mathbf{x}, t)\, \eta_\perp(\mathbf{x}', t') \rangle = 2\lambda_\perp \delta(t - t')\, \nabla_\perp^2 \delta(\mathbf{x} - \mathbf{x}') \tag{4a}$$

and

$$\langle n_\parallel(\mathbf{x}, t)\, \eta_\parallel(\mathbf{x}', t') \rangle = 2\lambda_\parallel \delta(t - t')\, \partial^2 \delta(\mathbf{x} - \mathbf{x}') \tag{4b}$$

Keeping in mind that τ played the role of temperature, the presence of $\tau_\perp$ and $\tau_{||}$ indicate that the system may be interpreted as being coupled to two thermal baths set at two different "temperatures." This somewhat unusual concept in fact makes good physical sense. Since the drive acts as an additional source of energy, jumps parallel to the field effectively experience a higher "temperature." Simple perturbation theory confirms this interpretation in the sense that, at the lowest order, a $\mathscr{E}^2$ term is *added* to the coefficient of $\partial^2\phi$. Thus, we are naturally led to the conclusion that, in the standard driven lattice gas, $\tau_\perp$ *will vanish first with* $\tau_{||} > 0$, as T is lowered to T_c. Meanwhile, the correlation of the noise is also modified and, given that detailed balance is typically absent in nonequilibrium systems, the ratio $\lambda_\perp/\lambda_{||}$ is no longer constrained by the fluctuation-dissipation theorem (FDT) to be $\tau_\perp/\tau_{||}$,[9,12] although both λ's remain positive at T_c. Using this ansatz and renormalization group techniques gives the upper critical dimension for this theory d_c to be 5, and a stable nontrivial fixed point and its associated exponents were obtained[9] in an expansion in powers of $(5-d)$. Subsequently, extensive simulation data largely confirmed these predictions.[13]

Given that there are two τ's, there is no theoretical reason for studying only the above case. Examining the remaining cases, two other results emerged.[9,10] If both τ's vanish simultaneously, d_c turns out to be 8 with a stable nontrivial fixed point in $d<8$. If, on the other hand, we let $\tau_{||} \to 0$ with $\tau_\perp > 0$, d_c becomes 4.5, with no *stable* fixed point in $d<d_c$. This result, along with the existence of a particular inhomogeneous solution to (3) at positive $\tau_{||}$ (in infinite system without free boundary conditions), was used as an argument for this transition being of *first order*.[9] However, up to this point, it has been unclear how these statements could be tested. Part of the purpose of this work is to introduce the microscopic dynamical model which most likely corresponds to this continuum theory.

While (3) is appropriate for systems with a nonvanishing steady-state current, it is clearly incorrect for systems driven with random fields or by two temperature baths. For those cases, it was argued[5] that only the ohmic term, $\mathscr{E}\partial\phi^2$, needs to be dropped. The reasoning is that, apart from this current, all the conditions for modifying (2) to arrive at (3) are still valid. In particular, the two-τ description, which allows only one τ to vanish at criticality, lends itself naturally to the two-temperature models.[6–8] In fact, in most simulation studies, either the higher temperature or the magnitude of the annealed random field is kept at infinity. Furthermore, having FDT-violating noise is entirely suitable, given the nonequilibrium nature of these models. Of course, the absence of $\mathscr{E}\partial\phi^2$ means that these theories no longer belong to the same universality class as the uniformly driven system.[5] Recent simulation data on $d=2$

lattices[8] are consistent with these predicions. Having $\mathscr{E}=0$ also implies that the differences between the three cases—letting $\tau_\perp$, $\tau_{||}$, or both vanish—are less dramatic, in the following sense. The first two cases are the same except for an exchange in the labeling transverse $\Leftrightarrow$ l, so that there is an obvious symmetry in $d=2$.[7] In the last case, if it is to model the two-temperature lattice gas, it would simply be an equilibrium system. On the other hand, there is no reason, in principle, to study *only* the FDT-respecting case, i.e., one with $g_\perp/g_{||}=\alpha_\times/\alpha_{||}=(\alpha_\perp/\alpha_{||})^{1/2}=\lambda_\perp/\lambda_{||}$. We are not aware of any investigations of such theories and will not address this issue further here.

Summarizing the status of continuum theories, most of the known varieties of driven diffusive systems can be retrieved from Eq. (3) by going to various regions of its parameter space. Restricting to the domain of positive α's and g's, one large region has not been realized by microscopic models: $\mathscr{E}\neq 0$ and $\tau_\perp>\tau_{||}$. In the next section, we present a lattice model which we argue should be the realization of this continuum theory. Renormalization group analyses lead to the conclusion that, as $\tau_{||}$ is lowered, this system will undergo a first-order transition from a homogeneous to a phase-segregated state. However, we will present simulation data that indicate that the transition of the lattice model, at least in $d=2$, is continuous.

3. MODEL DEFINITION

Our model consists of either a particle or a hole at each site of an $L\times L$ square lattice. Due to computational limitations, we restrict ourselves to $d=2$ here; but there are no conceptual difficulties with generalizations to hypercubic lattices in higher dimensions. A nearest-neighbor attractive interaction between the particles is present. In the Ising language, a particle/hole is a positive/negative spin σ, with ferromagnetic interactions. The lattice gas and spin languages will be used interchangeably. We lable a site by $\mathbf{x}\equiv(x_\perp, x_{||})$ and fix the boundary condition to be periodic. Choosing the interaction strength appropriately, we obtain the energy of a configuration

$$\mathscr{H}=-\sum\sigma(\mathbf{x})\,\sigma(\mathbf{y}) \tag{5}$$

where $\sigma=\pm 1$, and the sum is over nearest-neighbor sites $\mathbf{x}$ and $\mathbf{y}$.

Since we are interested in models whose dynamics conserve the number of particles, i.e., the total magnetization in the spin language, we choose to allow the system to evolve with Kawasaki spin exchanges between randomly chosen pairs of nearest-neighbor lattice sites. To define a general two-temperature driven system, we let spin exchanges in the "parallel" direction occur at an inverse temperature $\beta_{||}$ and those in the "transverse" direction occur at $\beta_\perp$. Additionally, we apply a driving field **E** which points in the "parallel" direction. As in the standard model, the field biases spin exchanges in favor of a positive (negative) spin moving along (against) the field direction. For simplicity, we choose the Monte Carlo rates of the microscopic dynamics to be Metropolis', so that the probability that a given pair of spins will exchange position in the perpendicular direction is given by the standard form[14]

$$W_\perp = \min\{1, \exp[-\beta_\perp \Delta\mathcal{H}]\} \tag{6a}$$

and, in the parallel direction, the effect of the field is incorporated through[2]

$$W_{||} = \min\{1, \exp[-\beta_{||}(\Delta\mathcal{H} + \varepsilon E)]\} \tag{6b}$$

where $\varepsilon = +1(-1)$ for an exchange attempting to move a positive spin against (along) the field.

Thus, the parameter space of our model is three dimensional: $(E, \beta_\perp, \beta_{||})$. In the subspace $E = 0$ lies the two-temperature model,[6-8] while the $\beta_\perp = \beta_{||}$ subspace refers to the standard driven lattice gas.[3] In principle, a coarse-graining procedure should lead us to the mesoscopic theory (3), with the parameters of the latter being functions of E, $\beta_\perp$, and $\beta_{||}$. In practice, these functions cannot be computed and we have little choice but to resort to making "educated guesses" at their general properties. Regarding this process as semiphenomenological and relying on the principles of universality, we make progress toward the understanding of these systems in the long-wavelength limit. In this spirit, we expect the following inequalities:

$$\tau_\perp(E=0, \beta_\perp, \beta_{||}) > \tau_{||}(E=0, \beta_\perp, \beta_{||}) \quad \text{if and } \textit{only if} \quad \beta_\perp < \beta_{||} \tag{7a}$$

$$\tau_\perp(E, \beta_\perp, \beta_{||}) < \tau_{||}(E, \beta_\perp, \beta_{||}) \quad \text{if} \quad E \neq 0 \text{ and } \beta_\perp = \beta_{||} \tag{7b}$$

$$\tau_\perp(E, \beta_\perp, \beta_{||}) > \tau_{||}(E, \beta_\perp, \beta_{||}) \quad \text{if} \quad E \neq 0 \text{ and } \beta_\perp \ll \beta_{||} \tag{8}$$

The first comes from previous studies[5] of the two-temperature model, i.e., the parameter τ is a monotonic measure of the temperature associated with that thermal bath. The second is, as discussed in the previous section, the effect of the drive on $\tau_{||}$. Combining these two arguments, we at last arrive at our proposed microscopic model for the untested region: $\mathscr{E} \neq 0$ with $\tau_\perp > \tau_{||}$. In particular, we expect that the effects of the drive on $\tau_{||}$ cannot reverse the inequality $\tau_\perp > \tau_{||}$, provided E is not too excessive and the transverse thermal bath is sufficiently "hotter" than the longitudinal one.

4. SIMULATION METHODS AND RESULTS

Since the total magnetization is conserved by the dynamics, the transition from disorder to order is displayed as phase separation, so that "generalized magnetizations"[2] must be used for order parameters. For simplicity, in this study we restricted the total magnetization to zero, corresponding to a half-filled lattice. Expecting *two* different types of ordering, transverse and longitudinal, we define

$$m_{\perp} \equiv C \left| \sum_{x_{\perp}} e^{i2\pi x_{\perp}/L} \sum_{x_{\parallel}} \sigma(x_{\perp}, x_{\parallel}) \right| \tag{9a}$$

and

$$m_{\parallel} \equiv C \left| \sum_{x_{\parallel}} e^{i2\pi x_{\parallel}/L} \sum_{x_{\perp}} \sigma(x_{\perp}, x_{\parallel}) \right| \tag{9b}$$

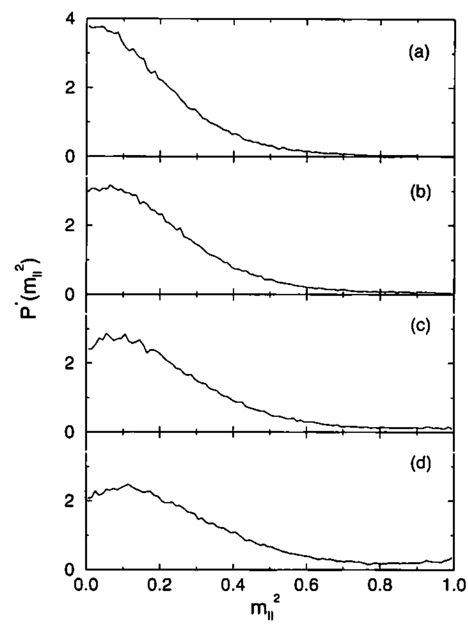

Fig. 2. Probability distributions of $m_{\parallel}^2$ for an $L=16$ system, with $E=4$, $\beta_{\perp}=0$, and $\beta_{\parallel}=0.52$ (a), 0.54 (b), 0.56 (c), and 0.58 (d).

where the normalization constant

$$C \equiv \{2[1 - \cos(2\pi/L)]\}^{1/2}/(4L) \tag{10}$$

is defined so that, for a perfectly transversely (longitudinally) ordered configuration, $m_\perp$ ($m_{||}$) is unity while the other one is zero. However, for random configurations, both m's *vanish*, in the thermodynamic limit. In this sense, they are suitable for measuring the degree of both types of order.

During the course of a simulation run we periodically measure $m_\perp^2$ and $m_{||}^2$ and then construct histograms of their values, which, when properly normalized, are the steady-state probability distributions $P(m_\perp^2)$ and $P(m_{||}^2)$. These histrograms contain considerable information and allow us to determine the phase of the system. Combined with a finite-size analysis, they also allow us to determine the location and order of phase transitions.

To obtain a qualitative picture of the phase diagram, we first monitored the simulations in real time. This also enabled us to observe the various processes, such as ordering dynamics. Then, long runs were carried out and quantitative measurements made. In these, the system sizes range from $L = 8$ to $L = 32$, while the length of the runs varied with system size,

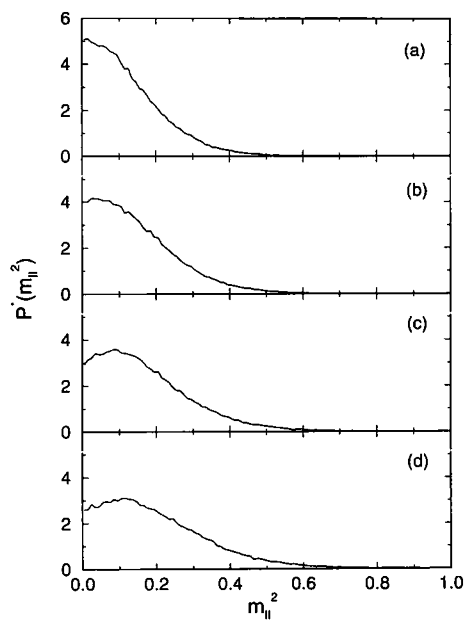

Fig. 3. Probability distributions of $m_{||}^2$ for an $L = 24$ system, with the same set of values for $(E, \beta_\perp, \beta_{||})$ as in Fig. 2.

typically from 4×10^5 Monte Carlo sweeps (MCS) for $L = 8$ systems to 3×10^6 MCS for $L = 32$ systems. The frequency of our measurement of the order parameters also depended on the size of our system and typically ranged from every 4 MCS for $L = 8$ systems to every 30 MCS for $L = 32$ systems. The data from the first 10% of each run (sometimes more) were discarded to allow the system to reach a steady state.

We begin our discussion of the simulation results with the study of the effects of the field E on the transition to *longitudinal* order. We start by restricting ourselves to the hottest "transverse" bath possible, i.e., $\beta_{\perp} = 0$. In the undriven case ($E = 0$), the transition occurring at $\beta_{\parallel c} \simeq 0.33$ is known to be continuous,[8] in agreement with field-theoretic predictions.[5] However, according to the theory, once a uniform driving field is applied, the transition should change to *first-order*.[9] To check this prediction, we imposed a field of strength $E = 4$ on lattices with $L = 16$, 24, and 32. Simulating these systems at various $\beta_{\parallel}$, we measured $P(m_{\parallel}^2)$. The results are shown in Figs. 2–4. In all three cases, we see that, for $\beta_{\parallel} < 0.54$, the distribution has a single peak, with a maximum at $m_{\parallel}^2 = 0$. However, at $\beta_{\parallel} \simeq 0.54$, this single maximum begins to shift, *continuously*, to positive values of $m_{\parallel}^2$. We interpret this behavior as a clear indication that the transition is *continuous* rather than first-order.

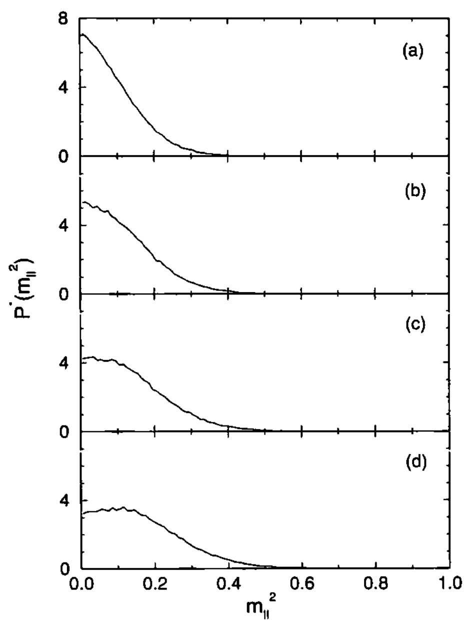

Fig. 4. Probability distributions of $m_{\parallel}^2$ for an $L = 32$ system, with the same set of values for $(E, \beta_{\perp}, \beta_{\parallel})$ as in Fig. 2.

At still larger values of $\beta_{\parallel}$, we observe a curious effect which *mimics a first-order transition.* As $\beta_{\parallel}$ is increased, a second peak develops in $P(m_{\parallel}^2)$, with maximum at $m_{\parallel}^2 = 1$. As $\beta_{\parallel}$ is raised further, the amplitude of the first peak decreases while the height of the new one increases. See Fig. 5. This process continues until the first peak disappears entirely, so that the distribution is again single-peaked, with maximum at unit $m_{\parallel}^2$. The presence of a double-peaked P is indicative of a first-order transition, if it were to hold for an infinitely large system. However, as shown in Fig. 6, if we hold $\beta_{\parallel}$ constant and increase L, then the amplitude of the second peak decreases. Alternatively, the temperature at which the two peaks are of equal height decreases with system size. We conjecture that this curious phenomenon disappears as $L \to \infty$, so that there is no true first-order transition.

Based on observations in real time, the mechanism responsible for this effect can be described as follows. For large β, particles (positive spins) tend to cluster due to the attractive interactions. On the other hand, the effect of the E field is to drive them out, through the "downstream" edge of the cluster. (For example, a part of such an edge is displayed in Fig. 7). Of course, on the average, we cannot expect this kind of interface to be

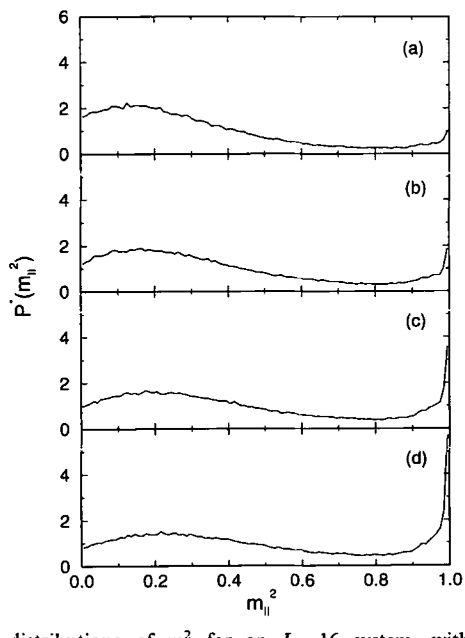

Fig. 5. Probability distributions of $m_{\parallel}^2$ for an $L=16$ system, with $E=4$, $\beta_{\perp}=0$, and $\beta_{\parallel}=0.60$ (a), 0.62 (b), 0.64 (c), and 0.68 (d).

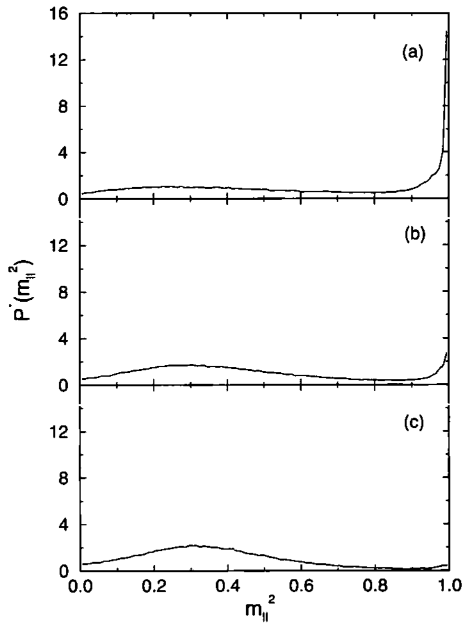

Fig. 6. Probability distributions of $m_{||}^2$ for systems with $E=4$, $\beta_\perp=0$, and $\beta_{||}=0.70$, for $L=16$ (a), 24 (b), and 32 (c).

microscopically flat, i.e., solidly filled on one side and totally empty on the other. However, once a flat one is form, then it can be quite stable, provided E is not excessively large. In particular, if a flat interface stretches across the system, it will typically persist for some time, often allowing all of the particles in the system to "fill in" behind it so that a perfectly ordered state is achieved. These are the configurations for which $m_{||}^2=1$ and are

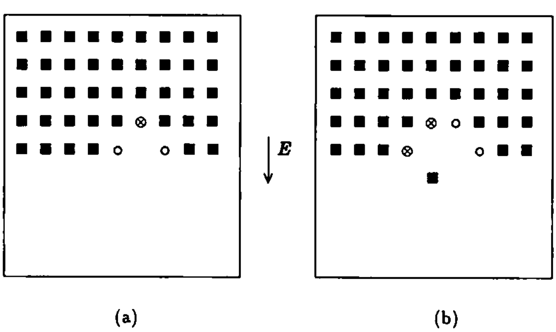

Fig. 7. An illustration of the effects of the drive on a "downstream" edge with a single hole (a). If $E>8$, a particle can be driven out (b), and an avalanche ensures.

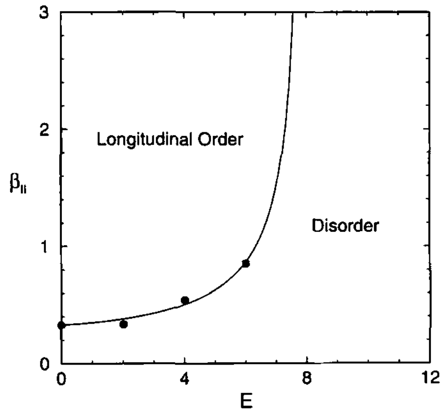

Fig. 8. Phase diagram of the *driven*, diffusive two-temperature Ising model in the $\beta_\perp = 0$ plane of the three-dimensional parameter space $(E, \beta_\perp, \beta_\parallel)$. The solid circles indicate the approximate location of the continuous transition as measured in the simulations. The curve is drawn as a guide to the eye, reflecting the simulation data, and the fact that no longitudinal order can exist for $E > 8$.

responsible for the appearance of the second peak in $P(m_\parallel^2)$. For a finite system, the probability for a flat interface to stretch across the lattice is nonzero. Thus, the amplitude of the second peak is presumably a delicate function of this probability and the lifetime of such an edge. However, it seems reasonable that both rates decrease with L, so that this phenomenon never occurs in the limit $L \to \infty$.

We have used the phrase "not excessively large" several times in connection with E. To be quantitative, it is trivial to see that no longitudinal order is possible if $E \geqslant 12$, since fields of this strength can tear any particle out of a perfectly ordered state. A more sophisticated argument[3] shows, however, that $E = 8$ is the generic limit of stability. In particular, consider a perfectly ordered state except for a single hole along the "downstream" edge (Fig. 7a), which may originate from a temperature fluctuation, an infinitesimal deviation from being half-filled, or an impurity. Now, it is clear that the two corner particles, denoted by $\bigcirc$, can be detached from the bulk if $E > 8$. Actually, $E > 4$ is enough to move another particle ($\otimes$) at this stage. But it cannot proceed beyond the next step unless $E > 8$. Once the corner particle is detached (Fig. 7b), two others ($\otimes$) are now susceptible to the smaller bound: $E > 4$, while two ($\bigcirc$) can be torn out by

[3] A version of the argument presented here, originally due to H. van Beijeren (private communication), was used to showed the limit of the ordered phase in a driven diffusive lattice gas with repulsive interparticle interactions.[15]

the larger $E>8$. Thus, an avalanche ensues and the ordered state is destroyed.

To test this argument and to explore the effects of other field strengths, we repeated the above study with $E=2$, 6, and 9. For $E=2$ and 6, we observed similar results as the $E=4$ case, except that the continuous transition occurred at different values of $\beta_{||}$. However, for $E=9$ no ordering was

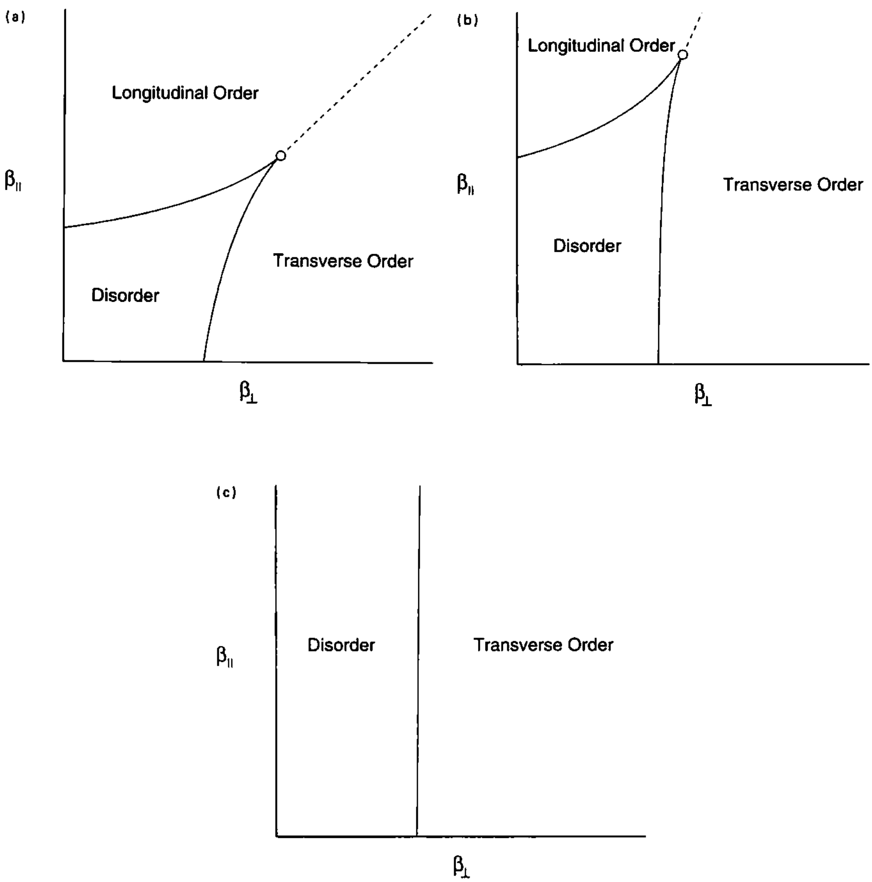

Fig. 9. Schematic phase diagram of the *driven*, diffusive two-temperature Ising model in the parameter space (E, $\beta_{\perp}$, $\beta_{||}$), presented in terms of various cross sections of constant E: (a) $E=0$, (b) $8>E$, and (c) $E>8$. The transitions from disorder to either ordered phase, indicated by solid lines, are continuous. However, the transition from disorder to *longitudinal* order, predicted to be first order by field theory,[9] may turn discontinuous in $d\geqslant 3$, in the manner of the three-state Potts model. The dashed line indicates first-order transitions and the open circle is the bicritical point.

ever observed, confirming $E=8$ as the limit of longitudinal order. Noting that the transition in the $E=0$ case occurs at about $\beta_{\|} \approx 0.33$, we may use these results to plot a phase diagram in the $(E, \beta_{\|}, \beta_{\perp}=0)$ plane, shown in Fig. 8. That $\beta_{\|c}$ increases with E is to be expected, since E encourages transverse order, as evidenced by the behavior of the standard model.[2]

We have performed similar analyses for the transition to transverse order. As expected from theory, they show that that transition remains continuous. For example, if we vary $\beta_{\perp}$ while fixing $\beta_{\|}=0$ and $E>0$, we find $P(m_{\perp}^2)$ to have only one peak. For small $\beta_{\perp}$, the maximum is located at $m_{\perp}^2=0$. As it increases beyond a certain critical value $\beta_{\perp c}$, the location of this maximum shifts continuously from 0 to positive values. As $\beta_{\perp}$ a approaches infinity, these values increase monotonically toward unity. No second peak develops for any L. However, in contrast to the transition to longitudinal order, $\beta_{\perp c}$ *decreases* with increasing E for any $\beta_{\|}$. As $E \to \infty$, it reaches a value of about 1/3, which corresponds to the critical point in the standard model with infinite E.[2] That correspondence is due to the trivial fact that $\lim_{E \to \infty} E\beta_{\|} = \infty$ for any $\beta_{\|}>0$, so that the rate (6b) becomes $\min\{1, 0\}$ regardless of $\beta_{\|}$.

Finally, we turn to the question of how the drive affects the bicritical structure in the phase diagram of the two-temperature model. Our results, which are summarized in Fig. 9, indicate that the general features remain the same, provided $E<8$, i.e., two critical lines meet a third line of first-order transitions. The former mark the boundaries between the disordered phase and each of the two ordered phases, which are separated by the latter line. The only effect of the field is a shift in the longitudinal (transverse) critical line to larger $\beta_{\|}$ (smaller $\beta_{\perp}$). Of course, the location of the bicritical point is shifted similarly. As E approaches 8, the transverse line remains at positive $\beta_{\perp}$ while the longitudinal crticial line recedes to $\beta_{\|}=\infty$, confirming the lack of longitudinal order for $E \geqslant 8$. Finally, as noted above, in the limit $E \to \infty$, the phase diagram consists of a single straight critical line at $\beta_{\perp} \simeq 0.33$.

5. CONCLUSIONS AND OUTLOOK

Using Monte Carlo simulations, we studied the nonequilibrium steady states of an Ising lattice gas driven by a combination of a uniform external field (E) and being coupled to two thermal reservoirs at different temperatures ($\beta_{\|}^{-1}$ and $\beta_{\perp}^{-1}$). In previous studies, two planes in this three-dimensional parameter space have been mapped out, i.e., the $\beta_{\|}=\beta_{\perp}$ case[2] and the $E=0$ model.[7] For the general model, we find that when $E \geqslant 8$ there are only two phases: the disordered and the transverse ordered one, and that the transition between them is continuous. On the other hand, for $E < 8$, the bicritical structure on the $E = 0$ plane persists qualitatively. In particular, the transitions from the disordered state to both types of ordered states remain continuous. While this result is expected for the transition to transverse order, it is rather surprising for the transition to longitudinal order, which was conjectured[(9)] to be first-order through a renormalization group analysis. However, this type of disagreement between field-theoretic results from the ε-expansion and simulation data is not novel. A well-known example is the Potts model.[4] In $d = 2$, the phase transitions in both the three- and four-state models are known to be continuous, while field-theoretic techniques find none other than first-order transitions. Of course, in that case, it was argued that there is no genuine contradiction, since $d = 2$ is "quite far" from the upper critical dimension of 6. The general belief is that the effects of the long-wavelength fluctuations are more pronounced in lower dimensions, so that the susceptibility diverges and a second-order transition is induced. It is surely sorthwhile to explore if the same mechanism is at play in our case.

Apart from this issue, many others remains to be investigated. For example, the universality classes of all the continuous transitions could be established through careful finite-size analyses. We could test the expectation that the transitions to transverse order will remain in the class of the standard driven model.[(9)] For transitions to longitudinal order, any result will be "new," since there are no predictions or measurements of those critical properties at present. Similarly, the neighborhood of the bicritical point will yield novel information, as it will have a *nonequilibrium* character. A detailed study of the mechanism leading to the states with perfect longitudinal order and the properties of the associated second peak in $P(m_{\parallel}^2)$ would be most interesting. Further, there may be more exotic transitions if we study systems with nonzero total magnetization, since the ordered state is likely to drift with a constant velocity. Beyond such "steady-state" phenomena, it is natural to study the behavior after a quench, e.g., nucleation, growth of ordered domains, and dynamic scaling. Clearly, this model presents us with many rich possibilities.

ACKNOWLEDGMENTS

We thank B. Schmittmann and Z. Rácz for stimulating discussions. This research is supported in part by grants from the National Science Foundation through the Division of Materials Research and the Jeffress Memorial Trust.

[4] For a review of the equilibrium Potts model see, e.g., ref. 16.

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